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THE ROLE OF RESONANCES IN RARE K DECAYS *

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ABSTRACT

The decays $K \to \pi l^+ l^-$, $K \to \pi\gamma\gamma$ and $K \to \pi\nu\bar{\nu}$ are investigated using the higher order terms of the chiral perturbation theory. The counterterms induced by strong, weak and electromagnetic interactions are determined assuming the resonance exchange.

The chiral perturbation theory (CHPT) offers an useful framework to describe K decays[1−15]. The role of resonances in CHPT has been much better understood during the last few years[5−12]. We discuss the rare decays $K \to \pi l^+ l^-$, $K_L \to \pi^0\gamma\gamma$ and $K \to \pi\nu\bar{\nu}$. The first two decays obtain contributions of higher order terms in momentum of CHPT[1−4]. The $K \to \pi\nu\bar{\nu}$ decays are suppressed due to the GIM mechanism and the dominant contribution comes from the short distance dynamics. We analyse their long distance contributions. The $K^+ \to \pi^+\nu\bar{\nu}$ decay amplitude obtains the leading contribution of $O(p^4)$, while the $K_{L,S} \to \pi^0\nu\bar{\nu}$ amplitude obtains the contribution of $O(p^2)$ chiral Lagrangians.

At the lowest order in momentum $O(p^2)$ the strong chiral Lagrangian is

$$\mathcal{L}_s^2 \;=\; \frac{f^2}{4}\{Tr(D_\mu U^\dagger D^\mu U\} + Tr(\chi U^\dagger + U\chi^\dagger)\}, \tag{1}$$

where $U = \frac{-i\sqrt{2}}{f}\phi$, $f \simeq f_\pi = 0.093$ GeV is the pion decay constant and ϕ is a pseudoscalar meson matrix[1]. The covariant derivative is given by $D_\mu U = \partial_\mu U + iUl_\mu - ir_\mu U$, l_μ and r_μ are external gauge field sources. The explicit chiral symmetry breaking induced by the electroweak currents of the standard model corresponds to the following choice:

$$r_\mu \;=\; eQ[A_\mu - tan\theta_W Z_\mu], \tag{2}$$

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$$l_\mu = eQ[A_\mu - tan\theta_W Z_\mu] + \frac{e}{sin\theta_W} Q_L^{(3)} Z_\mu + \frac{e}{\sqrt{2}sin\theta_W}[Q_L^{(+)}W_\mu^+ + Q_L^{(-)}W_\mu^-]. \quad (3)$$

Here Q's are the electroweak matrices[1−4,12]. The matrix χ takes into account the explicit breaking due to the quark masses in the underlying QCD Lagrangian[13]. The strong chiral Lagrangian at $O(p^4)$ has 10 phenomenological parameters which can be determined by low energy phenomenology[13]. The authors of ref.[6] have considered the resonance contribution to the coupling constants of this $O(p^4)$ effective strong chiral Lagrangian. They have found that the resonance exchange can saturate the finite part of the conterterms.

In the weak sector at $O(p^4)$, by imposing $SU(3)_L \times SU(3)_R$ chiral symmetry, one introduces many undetermined couplings[7,14,15]. Due to the lack of experimental data additional assumptions about the weak Lagrangian are necessary in order to fix the unknown couplings. There are two procedures available: the factorization model and the weak deformation model[1−4,7]. The factorization model [7−12] relies on the charged weak current $J_\mu = \delta S_2/\delta l_\mu + \delta S_4/\delta l_\mu + \delta S_6/\delta l_\mu + \cdots$, where S_i denotes the effective action of the order p^i. The effective weak Lagrangian is

$$\mathcal{L}_w = 4G_8 Tr(\lambda_6 J_\mu J^\mu), \quad (4)$$

with $G_8 = \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 g_8$ defined in ref.[1]. From $K \to \pi\pi$ it was found that $|g_8| = 5.1$. Both models can be formulated without any reference to resonances. Since the renormalized part of the strong couplings in chiral Lagrangian at $O(p^4)$ can be explained by resonance exchange[6], it is reasonable to apply the same procedure to the weak Lagrangian at $O(p^4)$[7].

$K \to \pi\gamma^*$

It was shown[1−4] that in the chiral perturbation theory at $O(p^2)$ $K \to \pi\gamma^*$ transitions are forbidden for a virtual photon $\gamma^*(q)$ for any value of q^2. Combining the contributions coming from one-loop and counterterms, induced by strong, weak and electromagnetic interactions[1−4], the amplitudes for $K^+ \to \pi^+\gamma^*$ and $K_S \to \pi^0\gamma^*$ can be written as

$$\mathcal{A}(K^+ \to \pi^+\gamma^*) = \frac{G_8 e}{(4\pi)^2} q^2 (W_+ + \Phi_K + \Phi_\pi)(q^2)\epsilon^\mu (p' + p)_\mu, \quad (5)$$

$$\mathcal{A}(K_S^0 \to \pi^0\gamma^*) = \frac{G_8 e}{(4\pi)^2} q^2 (W_S + 2\Phi_K)(q^2)\epsilon^\mu (p' + p)_\mu, \quad (6)$$

where p and p' are pion's and kaon's momenta. The loop contributions Φ_K and Φ_π are determined in ref.[1−3]. The couplings $W_+ = -(16\pi^2/3)(W_1^r + 2W_2^r - 12L_9^r) + (1/3)log(\mu^2/m_K m_\pi)$ and $W_S = -(16\pi^2/3)(W_1^r - W_2^r) + (1/3)log(\mu^2/m_K^2)$, where $W_{1,2}^r$ have been defined in ref.[2−4]. Using the equations of motion for resonances, the authors of ref.[6] have found that the finite parts of L_9^r, L_{10}^r and H_1^r get contributions from vector and axial-vector reconances:

$$L_9^V = \frac{F_V G_V}{2M_V^2}, \; L_{10}^V = 2H_1^V = -\frac{F_V^2}{4M_V^2}, \; L_9^A = 0, \; L_{10}^A = 2H_1^A = \frac{F_A^2}{4M_A^2}, \quad (7)$$

where M_V and M_A are the octet masses of vector and axial-vector mesons. The octet couplings $|F_V| = 0.154$ GeV $|G_V| = 0.069$ GeV are determined from the decay rates $\rho \to l^+l^-$ and

$\rho \to 2\pi$ respectively, while $F_A = 0.128$ GeV was determined in ref.[6]. We use $L_9^r = 6.9 \times 10^{-3}$, $L_{10}^r = -6.0 \times 10^{-3}$ and $H_1^r = 7.0 \times 10^{-3}$ (set A) obtained in ref.[6]. We also use the numerical values $L_9^r = 7.0 \times 10^{-3}$, $L_{10}^r = -5.9 \times 10^{-3}$ and $H_1^r = -4.7 \times 10^{-3}$ (set B), and $L_9^r = 5.8 \times 10^{-3}$, $L_{10}^r = -5.1 \times 10^{-3}$, $H_1^r = -2.4 \times 10^{-3}$ (set C), calculated in the extended Nambu and Jona-Lasinio model[16]. Among three possible decays of $K \to \pi e^+ e^-$, only the decay rate of $K^+ \to \pi^+ e^+ e^-$ has been measured. The branching ratio $BR(K^+ \to \pi^+ e^+ e^-) = (2.99 \pm 0.22) \times 10^{-7}$ from Brookhaven experiment gives the solution[17] $W_+ = 0.89^{+0.24}_{-0.14}$ from a fit to the high-q^2 spectrum. The solution extracted from the measured decay rate in the same experiment corresponds to the value $W_+ = 1.2^{+0.4}_{-0.5}$. Using the weak deformation model[1−4,7], it was found[6,10] $W_1^r = 4(L_9^r + L_{10}^r + 2H_1^r)$ and $W_2^r = 4L_9^r$, leading to $W_+^{W,A} = -5.01$, $W_S^{W,A} = 4.50$, $W_+^{W,B} = 3.91$, $W_S^{W,B} = 3.49$, and $W_+^{W,C} = 2.80$, $W_S^{W,C} = 2.38$. Neither of them satisfies the experimental result. If the factorization model is used[7,8,10], $W_1^r = 8(L_9^r + L_{10}^r + 2H_1^r)$ and $W_2^r = 8L_9^r$, leading to $W_+^{F,A} = -4.87$, $W_S^{F,A} = 8.67$, $W_+^{F,B} = 2.69$, $W_S^{F,B} = 6.69$, and $W_+^{F,C} = 1.22$, $W_S^{F,C} = 4.46$. It means that the factorization approach, with the couplings C, can reproduce the experimental result. However, $W_{1,2}^W$, obtained with the help of weak deformation approach, fulfills the condition for divergent parts of amplitudes[2,7], what $W_{1,2}^F$ does not contain.

The decay $K_L \to \pi^0 e^+ e^-$ is being investigated as a signal of direct $\Delta S = 1$ CP violation. In addition to a CP conserving process, which proceeds through two photon exchanges, there are two kinds of the CP violating decay[5]: one proportional to the well known parameter ϵ and the other direct CP violating effect. From our analyses[10], we calculate $BR(K_L \to \pi^0 e^+ e^-) = 1.15 \times 10^{-10}$, for $W_S^{F,C} = 4.46$, close to the experimental upper limit 4.3×10^{-9}, a result from[18], and 5.5×10^{-9} obtained by[19]. We support the idea[5] that the existence of direct CP violating term in the branching ratio $BR(K_L \to \pi^0 e^+ e^-)$ cannot be seen without observing $BR(K_S \to \pi^0 e^+ e^-)$.

$K \to \pi\gamma\gamma$

There are two measurements of the branching ratio: $BR(K_L \to \pi^0\gamma\gamma) = (1.70 \pm 0.3) \times 10^{-6}$ (NA31 result)[20] and $BR(K_L \to \pi^0\gamma\gamma) = (1.86 \pm 0.60 \pm 0.60) \times 10^{-6}$ (E731)[21]. The leading order contribution in CHPT ($O(p^4)$ order)[2] comes from the loops. The rate is then $BR(K_L \to \pi^0\gamma\gamma) = 0.67 \times 10^{-6}$. The corrections coming from physical intermediate states such as two pions were calculated ($O(p^6)$ order)[22]. In order to create a resonance exchange at $O(p^6)$, we follow the proposal of ref.[4] and we write the strong Lagrangian keeping the terms leading in $\frac{1}{N_c}$

$$\mathcal{L}_s^R = C_1^R F_{\mu\nu} F^{\mu\nu} Tr(Q^2 \partial_\lambda U^\dagger \partial^\lambda U) + C_2^R F_{\mu\alpha} F^{\mu\beta} Tr(Q^2 \partial^\alpha U^\dagger \partial_\beta U). \tag{8}$$

The $F_{\mu\nu}$ is the electromagnetic field strength tensor and the couplings $C_{1,2}^R$ are saturated by resonance exchange. We use the decay amplitudes for $V \to P\gamma$:

$$\mathcal{L}(V \to P\gamma) = ieG_{VP\gamma}\epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} Tr(V^\rho \{\partial^\sigma U, Q\}). \tag{9}$$

Using the nonet assumption for vector mesons, the ideal mixing, and the data for $V \to P\gamma$, we

calculate $|G_{VP\gamma}| = 7.7 \times 10^{-2}$. Eliminating vector mesons one easily derives

$$C_1^V = -\frac{1}{2}C_2^V = \frac{2e^2 G_{VP\gamma}}{M_V^2} = 3.92 \times 10^{-2} GeV^{-2}. \tag{10}$$

We have shown[9,11] that the contribution of scalar and tensor mesons is one order of magnitude smaller than this one. The amplitude for $K \to \pi\gamma\gamma$ can be decomposed into

$$\begin{aligned}\mathcal{A}(K(k) \to \pi(p)\gamma(q_1)\gamma(q_2)) &= \epsilon_\mu(q_1)\epsilon_\nu(q_2)[\frac{A(y,z)}{m_K^2}(q_2^\mu q_1^\nu - q_2 \cdot q_1 g^{\mu\nu}) \\ +2\frac{B(y,z)}{m_K^4}(-k\cdot q_1 k \cdot q_2 k^\mu k^\nu &- q_2 \cdot q_1 k^\mu k^\nu + k \cdot q_1 q_2^\mu k^\nu + k\cdot q_2 q_1^\mu k^\nu)],\end{aligned} \tag{11}$$

where A and B are functions of the Dalitz variables $y = (k\cdot(q_1 - q_2)/m_K^2)$ and $z = (q_1+q_2)^2/m_K^2)$. In ref.[4] the weak deformation approach was discussed for the calculation of the vector meson resonances. Using the factorization approach we derive

$$A^R(y,z) = \frac{8G_8 m_K^4 \alpha\pi}{9}(1+g(\theta))[4C_1^R(1-z+r_\pi^2) - C_2^R(1+z-r_\pi^2)], \tag{12}$$

$$B^R(y,z) = \frac{16G_8 m_K^4 \alpha\pi}{9}(1+g(\theta))C_2^R. \tag{13}$$

The function $g(\theta) = [m_K^2/(m_\eta^2 - m_K^2)](c-\sqrt{2}s)(c+2\sqrt{2}s) + [m_K^2/(m_{\eta'}^2 - m_K^2)](s+\sqrt{2}c)(s-2\sqrt{2}c)$ ($c \equiv cos\theta$, $s \equiv sin\theta$), is obtained by taking into account the mixing of the η and η' as usual[11]. In the large N_c limit $\theta \simeq -22^0$. Combining the resonance exchange with the contribution of loops at $O(p^4)$, the $O(p^6)$ unitarity corrections[22], we calculate $BR(K_L \to \pi^0\gamma\gamma) = 9.38 \times 10^{-7}$. Without resonances this braching ratio was found to be $BR(K_L \to \pi^0\gamma\gamma) = 8.38 \times 10^{-7}$. The invariant mass spectrum of the two photons is rather well reproduced in our approach[11].

The $K^+ \to \pi^+\gamma\gamma$ decay obtain the $O(p^4)$ counterterms contribution $\hat{c} = 32\pi^2/3[12(L_9 + l_{10}) - W_1 - 2W_2 - 2W_4]$ defined in ref.[1−4]. The weak deformation model[4] gives $\hat{c} = 0$, while the factorization model (set C) leads to $\hat{c} = -1.14$. The resonance exchange at $O(p^6)$ order gives the very small contribution to the amplitude

$$A(y,z)^{(6)} = \frac{16G_8 m_K^4 \alpha\pi}{9} C_1^R(1-z+r_\pi^2). \tag{14}$$

The branching ratio is found to be $BR(K^+ \to \pi^+\gamma\gamma) = 4.19 \times 10^{-7}$.

$K \to \pi\nu\bar{\nu}$

The $K \to \pi\nu\bar{\nu}$ decay amplitude is dominated by the short distance loop diagrams, due to the explicit dependence of the heavy quark mass. The $K^+ \to \pi^+\nu\bar{\nu}$ decay amplitude can be written as

$$\begin{aligned}\mathcal{A}(K^+ \to \pi^+\nu\bar{\nu}) &= \frac{G_F}{\sqrt{2}}\frac{\alpha f_+}{2\pi sin^2\theta_W}[V_{ts}^* V_{td}\xi_t(m_t^2/M_W^2) + V_{cs}^* V_{cd}\xi_c(m_t^2/M_W^2) \\ &+ V_{us}^* V_{ud}\xi_{LD}](k+p)^\mu \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_{\bar{\nu}})\end{aligned} \tag{15}$$

where f_+ is the form factor in $\bar{K}^0 \to \pi^+ e\bar{\nu}$ decay and k and p are K and π meson's momenta respectively. The decay amplitude $K^+ \to \pi^+ Z^0 \to \pi^+ \nu\bar{\nu}$ vanishes at $O(p^2)$[23–24]. The loop contributions were found to be roughly of order 10^{-7} smaller than that of short distance contributions[24]. We determine the long distance con tribution ξ_{LD} using the $O(p^4)$ chiral Lagrangian and assuming the factorization approach for the weak interactions

$$\xi_{LD} = \kappa\{8m_K^2 L_5^r(2sin^2\theta_W - 1) + q^2[L_9^r(2sin^2\theta_W - 1) + \frac{4}{3}L_{10}^r sin^2\theta_W + H_1^r(\frac{8}{3}sin^2\theta_W - 4)]\}, \quad (16)$$

with $\kappa = 4\pi^2 g_8/\sqrt{2}M_Z^2 cos^2\theta_W$. We calculate the branching ratio $BR(K^+ \to \pi^+\nu\bar{\nu})_{LD}^A = 0.17 \times 10^{-13}$, $BR(K^+ \to \pi^+\nu\bar{\nu})_{LD}^B = 0.29 \times 10^{-13}$ and $BR(K^+ \to \pi^+\nu\bar{\nu})_{LD}^C = 0.40 \times 10^{-13}$, using L_9^r, L_{10}^r and H_1^r from fits A, B and C. This is of order 10^{-3} smaller than the short distance contributions[25]. The decay $K_{S,L} \to \pi^0\nu\bar{\nu}$ has the leading contribution of $O(p^2)$ in CHPT. However, this leads to the branching ratio $BR(K_L \to \pi^0\nu\bar{\nu})_{LD} = 4.1 \times 10^{-18}$ and $BR(K_S \to \pi^0\nu\bar{\nu})_{LD} = 1.4 \times 10^{-15}$, what is entirely negligable comparing the leading short distance contribution[25].

We can summarize that the resonance saturation of the counterterms and the use of factorization approach can reproduce the experimental result for W_+ in $K^+ \to \pi^+e^+e^-$ decays, leading to the large value of W_S. It results in the prediction $BR(K_L \to \pi^0e^+e^-) = 1.15\times10^{-10}$. The resonance exchange, the loops of $O(p^4)$, and the unitarity corrections give $BR(K_L \to \pi^0\gamma\gamma) = 9.38\times10^{-7}$, when the factorization approach is applied. The long distance contributions in $K \to \pi\nu\bar{\nu}$ are much smaller than the leading short distance contributions.

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